WAVES IN FERROMAGNETIC FLUIDS

B. M. Berkovskii and V. G. Bashtovoi

UDC 532.511.538.6

It is shown that in viscous heat conductive ferromagnetic liquids with infinite conductivity under definite thermal conditions, slightly damped and undamped temperature and magnetohydrodynamic waves are propagated. The lengths and frequencies of the undamped waves are found. The physical mechanism of their excitation is discussed.

It has been established in [1] that slightly damped temperature and viscous low-frequency waves are propagated in a viscous heat conducting fluid in permanent gravitational and special temperature fields. It has been established successfully in [2] that the frequency band and lengths of waves which can be propagated with a small damping decrement is broadened in viscoelastic Maxwellian fluids. Because of the interaction between the magnetic moment per unit volume of fluid and the external magnetic field in ferromagnetic fluids, an improvement in the characteristics of the slightly damped temperature and magnetohydrodynamic waves should also be expected. The present paper studies this question.

Let us formulate the system of equations describing the nonstationary convective heat exchange processes in vsicous heat conducting ferromagnetic fluids with infinite conductivity in external magnetic fields:

$$\operatorname{div} \mathbf{B} = 0, \tag{1}$$

$$\partial_t \mathbf{B} = \operatorname{rot} [\mathbf{v} \times \mathbf{B}], \tag{2}$$

$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M}, \tag{3}$$

$$M = M(H,\ T), \tag{4}$$

$$\partial_t \rho + \operatorname{div} (\rho \mathbf{v}) = 0, \tag{5}$$

$$\rho [\partial_t \mathbf{v} + (\mathbf{v} \nabla) \mathbf{v}] = -\nabla p + [\operatorname{rot} \mathbf{H} \times \mathbf{B}] + \eta \Delta \mathbf{v} + \left(\zeta + \frac{\eta}{3}\right) \operatorname{grad} \operatorname{div} \mathbf{v} + \frac{1}{\mu_0} \nabla (\mathbf{MB}), \tag{6}$$

$$\rho c_p (\partial_t T + \mathbf{v} \nabla T) = \lambda \Delta T, \tag{7}$$

$$f(\rho,\ p,\ T) = 0. \tag{8}$$

Since we shall henceforth examine small amplitude waves, the transport coefficients were considered constant in deriving (1)-(8), and energy dissipation due to viscosity and the magnetocaloric effect was neglected.

A system of governing equations was obtained in [3] for a non-conducting incompressible fluid. Questions of the stability of the plane free surface of a ferromagnetic fluid in a magnetic field have been examined in [4-6].

Let us consider the propagation of small perturbations in a mechanical equilibrium background in a nonideal ferromagnetic fluid with infinite conductivity, in a homogeneous permanent magnetic field H_0 directed along the x axis, when a constant temperature gradient $\nabla T_0 = \vec{\gamma}$ directed along the y axis is present in the fluid.

A gradient M_0 is built up in the mechanical equilibrium state in the fluid because of the dependence M(T)

$$\nabla M_0 = \beta_2 \vec{\gamma}, \text{ where } \beta_2 = \left(\frac{\partial M}{\partial T}\right)_{H_0, T_0}.$$

Institute of Heat and Mass Exchange, Academy of Science BSSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 18, No. 5, pp. 899-904, May, 1970. Original article submitted June 25, 1969.

Following the known method of analysis [7], let us assume

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{h}, \quad \rho = \rho_0 + \rho', \quad p = p_0 + p', \quad T = T_0 + \theta, \quad \mathbf{v} \neq 0.$$

We shall conduct the further investigation in a first approximation, i.e., we shall neglect the products of the small quantities $\mathbf{h}$, ρ', p', θ, $\mathbf{v}$. Let us examine the one-dimensional problem and let us consider them to depend all on x and t. In this approximation

$$H = H_0 + h_x \text{ and } M = M_0 + \beta_1 h_x + \beta_2 \theta, \quad \beta_1 = \left(\frac{\partial M}{\partial H}\right)_{H_0, T_0}.$$

Neglecting the anisotropies of the medium, we assume the vectors B, M, and H to be parallel. Then

$$\mathbf{M} = \mathbf{M}_0 - \left(\frac{1}{H_0} - \frac{\beta_1}{M_0}\right) h_x \mathbf{M}_0 + \frac{M_0}{H_0}\mathbf{h} + \beta_2 \theta \frac{\mathbf{M}_0}{M_0}$$

and

$$\mathbf{B} = \mathbf{B}_0 - \left(\frac{1}{H_0} - \frac{\beta_1}{M_0}\right) h_x \mathbf{M}_0 + \left(\mu_0 + \frac{M_0}{H_0}\right)\mathbf{h} + \beta_2 \theta \frac{\mathbf{M}_0}{M_0}.$$

Limiting ourselves to the analysis of transverse velocity perturbations, let us depict the system (1)-(8) according to the components:

$$(\mu_0 + \beta_1)\,\partial_x h_x + \beta_2 \partial_x \theta + (\beta_2 \gamma / H_0)\, h_y = 0, \tag{9}$$

$$(\mu_0 + \beta_1)\,\partial_t h_x + \beta_2 \partial_t \theta + \gamma \beta_2 v_y = 0, \tag{10}$$

$$\partial_t h_y = H_0 \partial_x v_y, \tag{11}$$

$$\rho_0 \partial_t v_y = B_0 \partial_x h_y + \mu_0^{-1}\left[\beta_2(\mu_0 + 2\beta_1) + (M_0 + B_0)\,\sigma_2\right]\gamma h_x + \mu_0^{-1}\left[2\beta_2^2 + \sigma_1 (M_0 + B_0)\right]\gamma\theta + \eta \partial_x^2 v_y, \tag{12}$$

$$\partial_t \theta + \gamma v_y = \varkappa \partial_x^2 \theta, \tag{13}$$

where $\sigma_1 = (\partial^2 M / \partial T^2)_{H_0, T_0}$ and $\sigma_2 = (\partial^2 M / \partial H \partial T)_{H_0, T_0}$. Let us consider M to be a linear function of the temperature, as is valid for small changes. Then $\sigma_1 = 0$ and $\sigma_2 = \beta_1 \beta_2 / M_0$.

Perturbations of the Z-component of the magnetic field and of the velocity are propagated in the form of Alfven waves; hence, we shall not consider them.

To clarify the physical picture of the processes which occur, let us first consider an ideal fluid, i.e., let us put $\eta = \varkappa = 0$ in (12) and (13). Then the components h_y, v_y, and θ satisfy a wave equation of the form

$$\partial_t^2 \theta = U_\alpha^2 \partial_x^2 \theta - \omega_\beta^2 \theta, \tag{14}$$

where $\omega_\beta^2 = 2\gamma^2 \beta_2^2 / \mu_0 \rho_0$.

The propagation of transverse fluctuations in h_y and v_y in a conducting fluid placed in a magnetic field can be given a graphic physical interpretation [7]. It follows from (2) that

$$d_t (\mathbf{B}/\rho) = \left[(\mathbf{B}/\rho)\nabla\right]\mathbf{v},$$

where

$$d_t = \partial_t + (\mathbf{v}\nabla). \tag{15}$$

This is the condition of "freezing" the lines of force of magnetic field induction in the fluid, i.e., this means that each line of force is displaced together with the fluid particles thereon. Therefore, every transverse shift in the fluid causes a transverse shift in the line of forces which, because of the elasticity of the line of force, starts to be transmitted along it as a wave. Hence, each line of force of the magnetic field induction can be likened to a string along which small transverse oscillations are propagated. If we deal with a ferromagnetic medium, then a force [8] $\mu_0^{-1}\nabla(\mathrm{MB})$ acts per unit volume of this string, and we obtain the oscillations of a string subjected to external forces. Part of this force $\mu_0^{-1}\nabla(\mathrm{M_0 B_0})$ is equilibrated by the pressure gradient ∇p_0 in the state of mechanical equilibrium. The rest of this force is $f_m = 2\gamma\beta_2^2\mu_0^{-1}\theta$ and is a stimulating force. It follows from (7) that $d_t T = 0$ in an ideal fluid, which yields the condition of "freezing" of the isotherms in the fluid in the same sense as the "freezing" of the lines of magnetic field induction. Therefore, the isotherms and the lines of force of magnetic field induction are interrelated, and the former take on the elasticity of the latter. Hence, the propagation of transverse oscillations over the lines of force of magnetic field induction causes propagation of oscillations over the isotherms at the same velocity, as is described by (14). Therefore, we obtain the propagation of temperature waves. Because of the presence of the stimulating force, it can be expected that undamped waves will be propagated under definite conditions in a ferromagnetic fluid in the presence of viscosity and heat conductivity. The energy losses in this case will be made up because of the magnetic field energy.

Taking account of viscosity and heat conductivity, the temperature perturbation satisfies the following wave equation

$$\partial_t^2\theta = U_\alpha^2\partial_x^2\theta - \omega_\beta^2\theta + \varkappa\int\{\partial_x^2\partial_t^2\theta - U_\alpha^2\partial_x^4\theta + \omega_\beta^2(2-\alpha)\partial_x^2\theta - \nu\partial_x^4\partial_t\theta\}\,dt + \nu\partial_x^2\partial_t\theta, \tag{16}$$

where

$$\alpha = \mu_0(3 - \beta_1 H_0/M_0)/2(\mu_0 + \beta_1),$$

from which the dispersion equation

$$i\omega(i\omega - \nu k^2) + U_\alpha^2 k^2 + \omega_\beta^2 - \frac{\omega_\beta^2(1-\alpha)\varkappa k^2}{i\omega - \varkappa k^2} = 0\,, \tag{17}$$

follows for the solution in the form of the plane wave $\exp i(kx-\omega t)$.

In this case (7) yields $d_tT = \varkappa\Delta T$, and the isotherms are already not "frozen" in the sense understood earlier. Now a phase shift exists between the temperature changes and the fluid particle displacement, which is not 0 or π. A force

$$f_m = -2\mu_0^{-1}\beta_2^2\gamma\left[(1-\alpha)\theta + \gamma(2-\alpha)y\right],$$

now acts per unit volume of fluid, whose change is determined by the change in θ and y. Therefore, a phase shift exists between the change in force and the fluid particle displacement, for a definite value of which the work performed by this force can be positive. In fact, if the particle displacement varies according to the law

$$y = a\cos(kx - \omega t), \quad a > 0,$$

and the force according to the law

$$f = F\cos(kx - \omega t + \varphi) \equiv b\cos(kx - \omega t) + c\sin(kx - \omega t),$$

then the work of this force per unit time is $A = f\dot{y}$; $\dot{y} = \partial_t y$, and the mean work per period is

$$A = T^{-1}\int_0^T f\dot{y}\,dt = \frac{ac\omega}{2}.$$

If the phase shift were a multiple of π, c = 0, the mean work per period of this force would be zero, and for c > 0 which corresponds to a phase shift from π to 2π it is positive. In our case, if

$$y = a'e^{-k_2x}\cos(k_1x - \omega t) \equiv a\cos(k_1x - \omega t),$$

then it follows from (13) that

$$\theta = -\frac{\omega\gamma a}{\varkappa^2(k_1^2 - k_2^2)^2 + (\omega - 2\varkappa k_1k_2)}\left[\varkappa(k_1^2 - k_2^2)\sin(k_1x - \omega t) - (\omega - 2\varkappa k_1k_2)\cos(k_1x - \omega t)\right].$$

A viscosity force $f_B = \eta\partial_x^2 y$ acts per unit volume of fluid, and its mean work per period is

$$\bar{A}_B = T^{-1}\int_0^T f_B\dot{y}\,dt = -\frac{1}{2}\eta a^2\omega^2(k_1^2 - k_2^2),$$

where k_1 and k_2 are found from the solution of the dispersion equation (17) and only those values of k_1 and k_2 satisfying the inequality $|k_1| > |k_2|$ have any physical meaning since otherwise the mean work per period of the viscosity forces would be positive.

The mean work per period of the magnetic field force f_m equals

$$\bar{A}_m = \frac{1}{2}\rho_0\omega_\beta^2 a^2\omega^2(1-\alpha)\frac{\varkappa(k_1^2 - k_2^2)}{\varkappa^2(k_1^2 - k_2^2)^2 + (\omega - 2\varkappa k_1k_2)^2}$$

and is positive for $1-\alpha > 0$. If $B = \mu_0\mu_r H$, then $\alpha = (1/\mu_r) < 1$ for ferromagnets, and this condition is satisfied.

On the average, the viscosity and magnetic field forces perform the work

$$\bar{A}_m + \bar{A}_B = \frac{1}{2}\rho_0 a^2\omega^2 (k_1^2 - k_2^2)\left[\frac{\omega_\beta^2 (1-\alpha)\varkappa}{\varkappa^2 (k_1^2 - k_2^2)^2 + (\omega - 2\varkappa k_1 k_2)^2} - \nu\right], \quad (18)$$

per period.

1. For a negative value of this expression (the work of the magnetic field forces is less than the work of the viscosity forces), damped wave propagation should be expected.

2. When $\bar{A}_m + \bar{A}_B = 0$, waves with constant amplitude should be propagated.

3. When $\bar{A}_m + \bar{A}_B > 0$, the amplitude of the waves should grow.

Indeed, it is easy to obtain from the dispersion equation (17) (by putting $k = k_1 + ik_2$ and equating real and imaginary parts to zero) that

$$k_1 k_2 = -\frac{\omega (k_1^2 - k_2^2)}{2U_\alpha^2}\left[\frac{\omega_\beta^2 (1-\alpha)\varkappa}{\varkappa^2 (k_1^2 - k_2^2)^2 + (\omega - 2\varkappa k_1 k_2)^2} - \nu\right].$$

Since $k_1 = 2\pi/\lambda > 0$, then $k_2 > 0$ upon compliance with condition 1, which corresponds to a damped wave, $k_2 = 0$ upon compliance with condition 2, and the wave amplitude is constant, and $k_2 < 0$ upon compliance with condition 3, and the wave amplitude grows, as should have been expected from the preceding reasoning.

For $k_2 = 0$ the dispersion equation (17) yields

$$k_1^2 = -\frac{U_\alpha^2}{2\varkappa(\nu+\varkappa)} + \sqrt{\frac{U_\alpha^4}{4\varkappa^2(\nu+\varkappa)^2} - \frac{\omega_\beta^2}{\varkappa(\nu+\varkappa)}\left[1 - \frac{\varkappa}{\nu}(1-\alpha)\right]},$$

$$\omega^2 = U_\alpha^2 \frac{\varkappa}{\nu+\varkappa} k_1^2 + \omega_\beta^2 (2-\alpha)\frac{\varkappa}{\nu+\varkappa}.$$

It should be noted that taking account of the gravitational field force can strengthen the results obtained above, i.e., under definite conditions the gravitational field can also supply energy to the wave.

NOTATION

B, H	are the magnetic field induction and intensity vectors in the fluid;
M	is the magnetic moment vector per unit volume of fluid;
μ_0	is the magnetic permittivity of vacuum;
v	is the fluid velocity vector;
T, ρ, p	are the fluid temperature, density, and pressure;
σ	is the coefficient of fluid conductivity;
η, ζ	are the first and second fluid viscosities, respectively;
λ, $\varkappa$	are the coefficients of fluid heat and temperature conductivity;
ν	is the kinematic fluid viscosity;
ω	is the wave frequency;
k	is the wave number;
$\partial_x = \partial/\partial x$;	
$U_\alpha = B_0/\sqrt{\mu\rho_0}$	is the Alfven velocity.

LITERATURE CITED

1. A. V. Lykov and B. M. Berkovskii, Dokl. Akad. Nauk BSSR, 13, 4 (1969).
2. A. V. Lykov and B. M. Berkovskii, Inzh.-Fizich. Zh., 17, 5 (1969).
3. J. L. Neuringer and R. E. Rosensweig, Physics of Fluids, 7, 12 (1964).
4. M. D. Cowley and R. E. Rosensweig, J. Fluid Mech., 30, 4 (1967).
5. A. Gailitis, Magnitnaya Gidrodinamika, 1 (1969).
6. V. M. Zaitsev and M. I. Shliomis, Sixth Riga Conference of Magnetohydrodynamics; Abstracts of Reports [in Russian], 1, Riga (1968).
7. L. D. Landau and E. M. Lifshits, Electrodynamics of Continuous Media [in Russian], Fizmatgiz, Moscow (1959).
8. I. E. Tamm, Principles of the Theory of Electricity [in Russian], Nauka (1966).